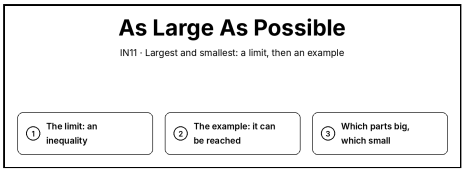
As Large As Possible: teacher script

IN11-V1 · IN11 Largest and smallest: a limit, then an example · teachers only. Without a screen, read each Say line and draw each Draw step on the board. About 117 seconds as a video.

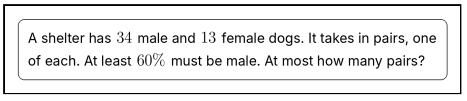
01



Say. Largest and smallest. The limit comes from an inequality. The example shows the limit can be reached. And think which parts should be big, and which small.

Draw. Write the title “As Large As Possible” and the steps: 1. The limit: an inequality; 2. The example: it can be reached; 3. Which parts big, which small.

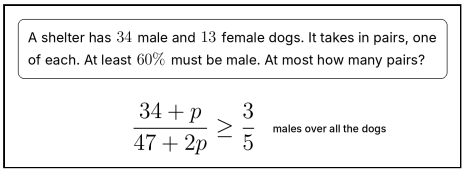
02



Say. A shelter has 34 male and 13 female dogs. It takes in pairs, one male and one female each, and wants at least 60 percent of its dogs to be male. At most how many pairs can it take in?

Draw. Clear the board. Write “A shelter has 34 male and 13 female dogs. It takes in pairs, one of each. At least 60\% must be male. At most how many pairs?” in a box at the top.

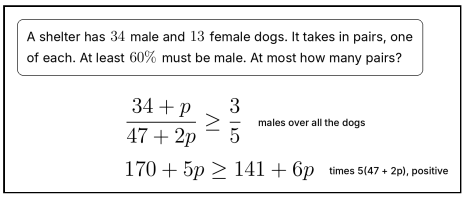
03



Say. With p pairs, it has 34 plus p males, out of 47 plus 2 p dogs: males over all the dogs, at least 3 fifths.

Draw. Write (34 + p)/(47 + 2p) \ge 3/5 on the next line. Beside it, write “males over all the dogs”.

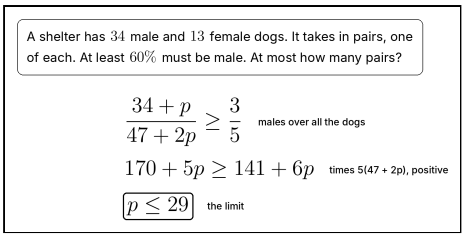
04



Say. Multiply both sides by 5 times 47 plus 2 p, which is positive, so keep the sign: 170 plus 5 p is at least 141 plus 6 p.

Draw. Write 170 + 5p \ge 141 + 6p on the next line. Beside it, write “times 5(47 + 2p), positive”.

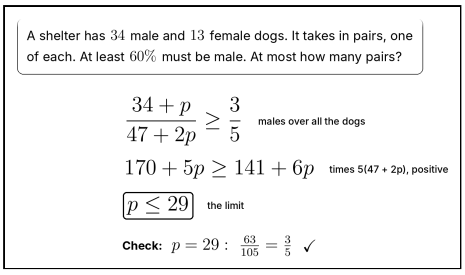
05



Say. So p is at most 29. That is the limit.

Draw. Write p \le 29 and box it on the next line. Beside it, write “the limit”.

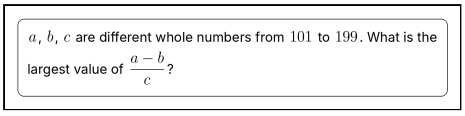
06



Say. And the example: with 29 pairs, 63 of 105 dogs are male, exactly 3 fifths. So the most is 29 pairs.

Draw. Below, write “Check: p = 29: \ 63/105 = 3/5” and tick it ✓.

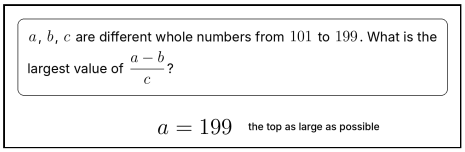
07



Say. a, b and c are different whole numbers from 101 to 199. What is the largest value of a minus b, over c?

Draw. Clear the board. Write “a, b, c are different whole numbers from 101 to 199. What is the largest value of (a − b)/c?” in a box at the top.

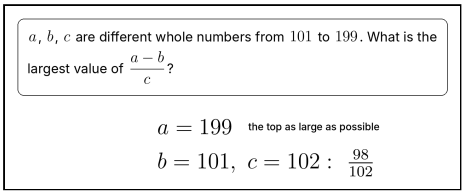
08



Say. Make the top as large as possible: a is 199, and b and c as small as possible.

Draw. Write a = 199 on the next line. Beside it, write “the top as large as possible”.

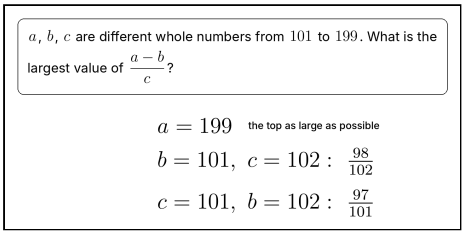
09



Say. But b and c must be different. If b is 101 and c is 102, we get 98 over 102.

Draw. Write b = 101,\ c = 102: \ 98/102 on the next line.

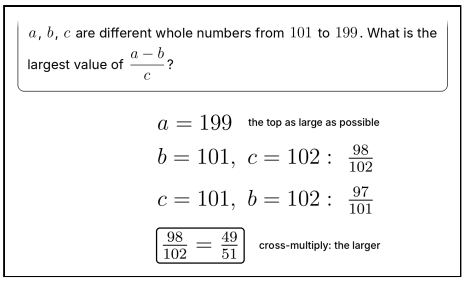
10



Say. If c is 101 and b is 102, we get 97 over 101.

Draw. Write c = 101,\ b = 102: \ 97/101 on the next line.

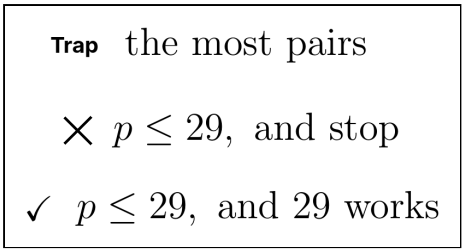
11



Say. Cross-multiply: 98 times 101 is more than 97 times 102. So the largest value is 49 over 51.

Draw. Write 98/102 = 49/51 and box it on the next line. Beside it, write “cross-multiply: the larger”.

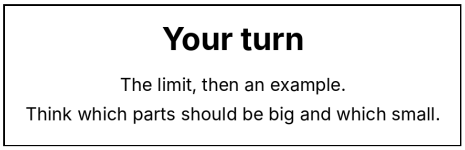
12



Say. Watch for the trap: don't stop at the inequality. Show that the limit can be reached: 29 works.

Draw. Clear the board. Write “Trap” and the most pairs. Below, write p \le 29, and stop and cross it out ×. Then write p \le 29, and 29 works with a tick ✓.

13



Say. Now it is your turn. Find the limit, then an example that reaches it.

Draw. Point to today’s activity: The limit, then an example. Think which parts should be big and which small.