As Large As Possible

IN11-V1 · storyboard · IN11 Largest and smallest: a limit, then an example. Read each picture with its sentences.

Picture 1

A shelter has 34 male and 13 female dogs. It takes in pairs, one of each. At least 60% must be male. At most how many pairs?

$$\frac{34+p}{47+2p} \geq \frac{3}{5}$$ males over all the dogs

$$170+5p \geq 141+6p$$ times 5(47 + 2p), positive

$$\boxed{p \leq 29}$$ the limit

Check: $p = 29: \ \frac{63}{105} = \frac{3}{5}$ ✓

1. A shelter has 34 male and 13 female dogs.
2. It takes in pairs, one male and one female each, and wants at least 60 percent of its dogs to be male.
3. At most how many pairs can it take in?
4. With p pairs, it has 34 plus p males, out of 47 plus 2 p dogs: males over all the dogs, at least 3 fifths.
5. Multiply both sides by 5 times 47 plus 2 p, which is positive, so keep the sign: 170 plus 5 p is at least 141 plus 6 p.
6. So p is at most 29.
7. That is the limit.
8. And the example: with 29 pairs, 63 of 105 dogs are male, exactly 3 fifths.
9. So the most is 29 pairs.

Picture 2

a, b, c are different whole numbers from 101 to 199. What is the largest value of $\dfrac{a-b}{c}$?

$a = 199$ the top as large as possible

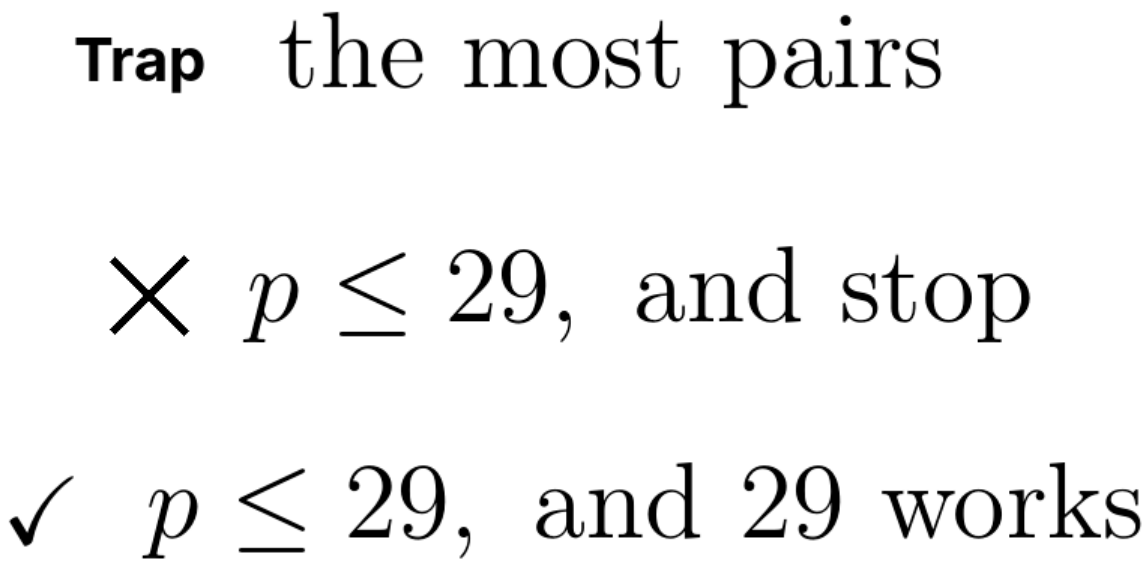
$b = 101,\ c = 102:\ \dfrac{98}{102}$

$c = 101,\ b = 102:\ \dfrac{97}{101}$

$\boxed{\dfrac{98}{102} = \dfrac{49}{51}}$ cross-multiply: the larger

10. a, b and c are different whole numbers from 101 to 199.
11. What is the largest value of a minus b, over c?
12. Make the top as large as possible: a is 199, and b and c as small as possible.
13. But b and c must be different.
14. If b is 101 and c is 102, we get 98 over 102.
15. If c is 101 and b is 102, we get 97 over 101.
16. Cross-multiply: 98 times 101 is more than 97 times 102.
17. So the largest value is 49 over 51.

Picture 3



18. Watch for the trap: don't stop at the inequality.
19. Show that the limit can be reached: 29 works.